CAMBRIDGE
International Education

# Cambridge International AS & A Level

**MATHEMATICS** **9709/52**

Paper 5 Probability & Statistics 1 **May/June 2024**

MARK SCHEME

Maximum Mark: 50

**Published**

This mark scheme is published as an aid to teachers and candidates, to indicate the requirements of the examination. It shows the basis on which Examiners were instructed to award marks. It does not indicate the details of the discussions that took place at an Examiners' meeting before marking began, which would have considered the acceptability of alternative answers.

Mark schemes should be read in conjunction with the question paper and the Principal Examiner Report for Teachers.

Cambridge International will not enter into discussions about these mark schemes.

Cambridge International is publishing the mark schemes for the May/June 2024 series for most Cambridge IGCSE, Cambridge International A and AS Level and Cambridge Pre-U components, and some Cambridge O Level components.



 **[Turn over**

**PUBLISHED**

## Generic Marking Principles

These general marking principles must be applied by all examiners when marking candidate answers. They should be applied alongside the specific content of the mark scheme or generic level descriptions for a question. Each question paper and mark scheme will also comply with these marking principles.

GENERIC MARKING PRINCIPLE 1:
Marks must be awarded in line with:

- the specific content of the mark scheme or the generic level descriptors for the question
- the specific skills defined in the mark scheme or in the generic level descriptors for the question
- the standard of response required by a candidate as exemplified by the standardisation scripts.

GENERIC MARKING PRINCIPLE 2:
Marks awarded are always **whole marks** (not half marks, or other fractions).

GENERIC MARKING PRINCIPLE 3:
Marks must be awarded **positively**:

- marks are awarded for correct/valid answers, as defined in the mark scheme. However, credit is given for valid answers which go beyond the scope of the syllabus and mark scheme, referring to your Team Leader as appropriate
- marks are awarded when candidates clearly demonstrate what they know and can do
- marks are not deducted for errors
- marks are not deducted for omissions
- answers should only be judged on the quality of spelling, punctuation and grammar when these features are specifically assessed by the question as indicated by the mark scheme. The meaning, however, should be unambiguous.

GENERIC MARKING PRINCIPLE 4:
Rules must be applied consistently, e.g. in situations where candidates have not followed instructions or in the application of generic level descriptors.

GENERIC MARKING PRINCIPLE 5:
Marks should be awarded using the full range of marks defined in the mark scheme for the question (however; the use of the full mark range may be limited according to the quality of the candidate responses seen).

GENERIC MARKING PRINCIPLE 6:
Marks awarded are based solely on the requirements as defined in the mark scheme. Marks should not be awarded with grade thresholds or grade descriptors in mind.

**Mathematics-Specific Marking Principles**

1 Unless a particular method has been specified in the question, full marks may be awarded for any correct method. However, if a calculation is required then no marks will be awarded for a scale drawing.

2 Unless specified in the question, non-integer answers may be given as fractions, decimals or in standard form. Ignore superfluous zeros, provided that the degree of accuracy is not affected.

3 Allow alternative conventions for notation if used consistently throughout the paper, e.g. commas being used as decimal points.

4 Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored (isw).

5 Where a candidate has misread a number or sign in the question and used that value consistently throughout, provided that number does not alter the difficulty or the method required, award all marks earned and deduct just 1 A or B mark for the misread.

6 Recovery within working is allowed, e.g. a notation error in the working where the following line of working makes the candidate's intent clear.

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 1(a) | $[(0.7)^6 \times 0.3] = 0.0353$ | **B1** | $\frac{352947}{10000000}$ or 0.03529… to at least 3sf. |
| | | **1** | |
| 1(b) | **Method 1** | | |
| | [P($X < 6$) =] $1 - 0.7^5$ | **M1** | $1 - 0.7^d$, $d = 5, 6$. |
| | = 0.832 | **A1** | Accept 0.83193 to at least 3sf.<br>If M0 scored, **SC B1** for 0.8319[3]. |
| | **Method 2** | | |
| | [P($X < 6$) =]<br>$0.3 + (0.3)(0.7) + (0.3)(0.7)^2 + (0.3)(0.7)^3 + (0.3)(0.7)^4$ | **(M1)** | $0.3 + (0.3)(0.7) + (0.3)(0.7)^2 + (0.3)(0.7)^3 + (0.3)(0.7)^4$ [+(0.3)(0.7 |
| | = 0.832 | **(A1)** | Accept 0.83193 to at least 3sf.<br>If M0 scored, **SC B1** for 0.8319[3]. |
| | | **2** | |
| 1(c) | $(0.7)^8 \times (0.3)^2 \times {}^9C_1$<br>or $(0.7)^8 \times (0.3) \times {}^9C_1 \times (0.3)$ | **M1** | $(0.7)^8 \times (0.3)^2 \times k$, $k$ a positive integer, 1 may be implied.<br>No addition/subtraction/additional terms. |
| | = 0.0467 | **A1** | |
| | | **2** | |

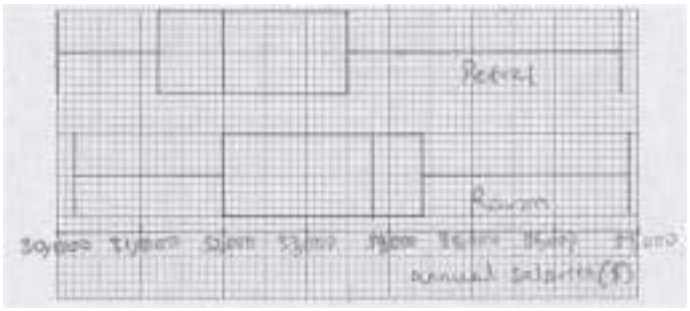
| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(a) | **Method 1** | | |
| | P(HR) + P(TR) + P(TBR)<br>$\frac{1}{3}\times\frac{4}{9}\ +\frac{2}{3}\times\frac{4}{9}\ +\frac{2}{3}\times\frac{5}{9}\times\frac{4}{8}$<br><br>or<br>P(HR) + (P(TRR) + P(TRB)) + P(TBR)<br>$\frac{1}{3}\times\frac{4}{9}+\left(\frac{2}{3}\times\frac{4}{9}\times\frac{3}{8}+\frac{2}{3}\times\frac{4}{9}\times\frac{5}{8}\right)+\frac{2}{3}\times\frac{5}{9}\times\frac{4}{8}$ | **B1** | Two of the calculations for P(HR), P(TBR), either P(TR) or P(TRR) + P(TRB) unsimplified, ignore any identification.<br>Condone $\frac{4}{8}=\frac{1}{2}$ in the unsimplified calculation.<br>Condone use of tree diagram to show calculation if values correct at end. |
| | | **M1** | Values of all correct identified scenarios added.<br>Correct branches may be identified on the tree diagram. |
| | $\left[=\frac{4}{27}+\frac{8}{27}+\frac{5}{27}\right]=\frac{17}{27}$ | **A1** | 0.6296…, 0.630<br>If M0 scored **SC B1** for acceptable answers, WWW. |
| | **Method 2** | | |
| | $1 - P(HB) - P(TBB) = 1-\left(\frac{1}{3}\times\frac{5}{9}+\frac{2}{3}\times\frac{5}{9}\times\frac{4}{8}\right)=\left[1-\frac{5}{27}-\frac{5}{27}\right]$ | **(B1)** | One calculation of P(HB), P(TBB), unsimplified, ignore any identification.<br>1 – probability must be seen.<br>Condone use of tree diagram to show calculation if values correct at end. |
| | | **(M1)** | 1 – values of two correct identified scenarios subtracted.<br>Correct branches may be identified on the tree diagram. |
| | $=\frac{17}{27}$ | **(A1)** | 0.6296…, 0.630<br>If M0 scored **SC** B1 for acceptable answers, WWW. |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(a) | **Method 3** | | |
| | P(HR) + P(T, (1 – no R)) =<br><br>$\frac{1}{3}\times\frac{4}{9}+\frac{2}{3}\left(1-\left(\frac{5}{9}\times\frac{4}{8}\right)\right)=$<br><br>$\left[\frac{4}{27}+\frac{2}{3}\left(1-\frac{20}{27}\right)\right]$ | **(B1)**<br><br>**(M1)** | Calculation for P(T, (1 – no R)) seen unsimplified.<br>Condone use of tree diagram to show calculation if values correct at end.<br><br>Values of two correct identified scenarios added.<br>Correct branches may be identified on the tree diagram. |
| | $=\frac{17}{27}$ | **(A1)** | 0.6296…, 0.630<br>If M0 scored **SC B1** for acceptable answers, WWW. |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 2(b) | **Method 1** | | |
| | $\left[\text{P(head} \mid \text{ no reds)} = \frac{\text{P(head} \cap \text{ no reds)}}{\text{P(no reds)}}\right]$<br>$= \frac{\frac{1}{3}\times\frac{5}{9}}{1-\frac{17}{27}} = \frac{\frac{5}{27}}{\frac{10}{27}} =$ | **M1** | $\frac{d}{1-their(\mathbf{a})}$ or $\frac{d}{1-\frac{17}{27}}$ or $\frac{d}{\frac{10}{27}}$, $0 < d < 1$.<br>Condone $\frac{10}{27} = 0.3704$ or more accurate. |
| | $= \frac{1}{2}$ | **A1** | OE<br>Condone $0.499[9\ldots]$. |
| | **Method 2** | | |
| | $\left[\text{P(head} \mid \text{ no reds)} = \frac{\text{P(head} \cap \text{ blue)}}{\text{P(HB)} + \text{P(TBB)}}\right]$<br>$= \frac{\frac{1}{3}\times\frac{5}{9}}{\frac{1}{3}\times\frac{5}{9}+\frac{2}{3}\times\frac{5}{9}\times\frac{4}{8}} = \frac{\frac{5}{27}}{\frac{10}{27}} =$ | **(M1)** | $\frac{d}{\frac{1}{3}\times\frac{5}{9}+\frac{2}{3}\times\frac{5}{9}\times\frac{4}{8}}$ or $\frac{d}{\frac{10}{27}}$, $0 < d < 1$.<br>Condone $\frac{10}{27} = 0.3704$ or more accurate. |
| | $= \frac{1}{2}$ | **(A1)** | OE<br>Condone $0.499[9\ldots]$. |
| | | **2** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 3(a) | $P(X>184) = P\left(Z > \frac{184-131}{54}\right)$ [= P( $Z > 0.9815$) ] | **M1** | Use of ±standardisation formula with 184, 131 and 54, no continuity correction.<br>Condone use of $\sigma^2$, $\sqrt{\sigma}$. |
| | 1 – 0.837 [= 0.163] | **M1** | Calculating the appropriate probability area (leading to their final answer). |
| | Percentage [= $0.163 \times 100$] = 16.3 | **A1** | AWRT |
| | | **3** | |
| 3(b) | [P($X < w$) = P($Z < \frac{w-131}{54}$) = 0.2]<br>$\frac{w-131}{54} = -0.842$ | **B1** | $-0.842 \leqslant z < -0.8415$ or $0.8415 < z \leqslant 0.842$ seen. |
| | | **M1** | Use of the ±standardisation formula with 131, 54, $w$ and a $z$-value (not 0.2, 0.8, 0.158, 0.508[0], 0.492[0], 0.7881, 0.2119, 0.5593, 0.4407). |
| | $w = 85.5$ | **A1** | $85.5 \leqslant p \leqslant 85.6$<br>Signs must be consistent to create a positive answer. |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(a) | Median = 32000 | **B1** | Clearly identified, e.g. Q2, med.<br>Accept 32 k. |
| | [UQ = 33500, LQ = 31200]<br>[IQR =] 33500 – 31200 | **M1** | 33300 ⩽ UQ ⩽ 33700 – 31100 ⩽ LQ ⩽ 31200<br>Implied if both quartile values are stated and an appropriate IQR is calculated accurately. |
| | = 2300 | **A1** | WWW<br>Ignore $ signs.<br>If M0 scored, **SC B1** for 2300 WWW.<br>If key ignored consistently:<br>B0 Median = 320<br>**SC M1** 325 ⩽ UQ ⩽ 335 – 311 ⩽ LQ ⩽ 312<br>**SC A1** 23. |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 4(b) | Box-and-whisker plot on provided grid<br>  <br><br>R: 30 200, 32 000, 33 800, 34 400, 36 900<br>P: 30 000, 31 200, 32 000, 33 500, 36 800 | **B1** | All five key values for *R* plotted accurately in standard format using a linear scale with at least three linear values. Labelled *R*.<br>Condone whiskers through box or at corners of boxes or extending $\frac{1}{2}$ square beyond limit.<br>Scale no less than 1 cm = \$1000.<br>Daylight rule applied to vertical lines of box. |
| | | **B1FT** | All five key values for *P*, FT from **(a)**, plotted accurately in standard format using a linear scale with at least three linear values. Labelled *P*.<br>Condone whiskers through box or at corners of boxes or extending $\frac{1}{2}$ square beyond limit.<br>Scale no less than 1 cm = \$1000.<br>Daylight rule applied to vertical lines of box. |
| | | **B1** | Whiskers not through box (condone $\frac{1}{2}$ square in box) for either, not drawn at corners of boxes. single linear scale for the diagram and labelled 'salaries' (OE) and \$.<br>If only one plot attempted, **SC B1** for meeting all the requirements above. |
| | | **3** | |
| 4(c) | Median because there is an extreme value (\$36 800) | **B1** | Do not accept 'values'.<br>Must identify median and reference either the extreme value (anomaly, outlier, 36 800) or the skew in context (e.g. concentrated in lower values, positive skew). |
| | | **1** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(a) | [$7 =] [$]5 + [$]2<br>[Probability =] $\frac{1}{5}\times\frac{2}{4}\times 2=\frac{1}{5}=0.2$<br>Or<br>[Probability =] 0.2 × 0.5 × 2 = 0.2 | **B1** | AG<br>Must include [$7], 5, 2 and link the probabilities to the appropriate value<br>$\frac{\left[{}^{1}C_{1}\right]\times {}^{2}C_{1}}{{}^{5}C_{2}}=0.2.$<br>$\frac{1}{5}\times\frac{2}{4}+\frac{2}{5}\times\frac{1}{4}$, not $\frac{1}{5}\times\frac{2}{4}+\frac{1}{5}\times\frac{2}{4}$ unless 5 and 2 and 2 and 5 seen in solution.<br>If all possibilities identified (e.g. outcome table), must be clearly labelled and terms fulfilling the condition identified. |
| | | **1** | |
| 5(b) | $x$ \| 2 \| 3 \| 4 \| 6 \| 7<br>P($X = x$) \| **0.1** \| **0.4** \| **0.1** \| **0.2** \| 0.2 | **B1** | Table with correct $x$ values and at least one further non-zero probability correct.<br>Condone extra $x$ values if probability stated as 0. |
| | | **B1** | Two more correct non-zero probabilities linked with correct outcomes. Accept probabilities not in table if clearly identified. |
| | | **B1** | All five probabilities correct. Accept probabilities not in table if clearly identified.<br>**SC B1** for four further non-zero probabilities adding to 0.8 if B1 max scored. |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 5(c) | [E($X$) = 0.1 × 2 + 0.4 × 3 + 0.1 × 4 + 0.2 × 6 + 0.2 × 7]<br>**0.2 + 1.2 + 0.4 + 1.2 + 1.4** [ = 4.4] | **M1** | Accept unsimplified expression. May be calculated in the variance, FT *their* table with **at least 5** probabilities, $0 < p < 1$, that sum to 1. FT acceptable at the bold partially evaluated stage. |
| | $[\text{Var}(X) = 0.1\times 2^2 + 0.4\times 3^2 + 0.1\times 4^2 + 0.2\times 6^2$<br>$+0.2\times 7^2 - 4.4^2$<br>$0.1\times 4 + 0.4\times 9 + 0.1\times 16 + 0.2\times 36 + 0.2\times 49 - 4.4^2$ | **M1** | Appropriate variance formula using *their* $(\text{E}(X))^2$ value. FT *their* table with **at least 4** probabilities, $0 < p < 1$, that may not sum to 1. FT acceptable at the bold partially evaluated stage.<br>Note: if table is correct, 22.6 – ($4.4^2$ or 19.36) implies this M1. |
| | = 3.24 | **A1** | CAO<br>$\frac{81}{25}$, $3\frac{6}{25}$ scores A0.<br>Only dependent upon previous M1 (M0 M1 A1 possible).<br>If M0 M0 scored, **SC B1** for 3.24 WWW. |
| | | **3** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(a) | Mean $[=110\times0.25]=27.5$<br>Variance $[=110\times0.25\times0.75]=20.625,\ \frac{165}{8}$ | **B1** | 27.5 and 20.625 (CAO) seen, allow unsimplified.<br>May be in standardisation formula<br>(4.541475… to at least 4sf or $\sqrt{\frac{165}{8}}$ or $\frac{\sqrt{330}}{4}$ implies correct variance).<br>Penalise incorrect identification, condone no identification. |
| | $P(X<22)=P\left(Z<\frac{21.5-27.5}{\sqrt{20.625}}\right)$ | **M1** | Substituting *their* 27.5 and *their* 20.625 into the ±standardising formula (any number for 21.5), not $\sigma^2$, not $\sqrt{\sigma}$. |
| | | **M1** | Using continuity correction 21.5 or 22.5 in *their* standardisation formula. |
| | $[P(Z<-1.3212)=1-\Phi(1.3212)]$<br>$1-0.9068=$ | **M1** | Appropriate probability area, from final process, must be a probability.<br>May be implied by a sketch of the required probability area.<br>Expect final answer < 0.5. |
| | 0.0932 | **A1** | $0.0932\leqslant p<0.09325$<br>If either M1 M1 not awarded for standardisation and/or M1 not awarded for finding probability area, **SC B1** $0.0932\leqslant p<0.09325$ WWW. |
| | | **5** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 6(b) | **Method 1** | | |
| | [1 – P(8, 9, 10) = ]<br>$1 - ({}^{10}C_8\, 0.85^8\, 0.15^2 + {}^{10}C_9\, 0.85^9 0.15^1 + 0.85^{10})$<br>[ = 1 – (0.275897 + 0.347425 + 0.196874)] | **M1** | One term ${}^{10}C_x\, (p)^x (1-p)^{10-x}$, $0 < p < 1$, $x \neq 0$ or 10. |
| | | **A1** | Correct unsimplified expression.<br>Condone omission of last bracket only. |
| | = 0.180 | **B1** | $0.1795 < p \leqslant 0.180$ |
| | **Method 2** | | |
| | [P(0, 1, 2, 3, 4, 5, 6, 7) = ]<br>$0.15^{10} + {}^{10}C_1\, 0.85 \times 0.15^9 + \ldots + {}^{10}C_7\, 0.85^7 0.15^3$ | **(M1)** | One term ${}^{10}C_x\, (p)^x (1-p)^{10-x}$, $0 < p < 1$, $x \neq 0$ or 10. |
| | | **(A1)** | Correct unsimplified expression. |
| | = 0.180 | **(B1)** | $0.1795 < p \leqslant 0.180$ |
| | | **3** | |
| 6(c) | $0.25 \times 0.6 \times 0.15 \times 6$ | **M1** | $0.25 \times 0.6 \times 0.15 \times k$, $k$ an integer > 1. |
| | 0.135, $\frac{27}{200}$ | **A1** | |
| | | **2** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(a) | $\left[\frac{10!}{2!4!} =\right]$ 75600 | **B1** | |
| | | **1** | |
| 7(b) | 4! × 3! | **M1** | 4! SOI in all terms leading to final answer.<br>Allow 24 if 4! = 24 is seen. |
| | | **M1** | Ignoring any values used to justify 4!.<br>Either 3! SOI in expression leading to final answer, or at least 6 distinct scenarios identified and added in expression leading to final answer.<br>Condone 3 distinct scenarios × 2.<br>Ignore repeated scenarios. |
| | | **A1** | 4! × 3!<br>Fully correct unsimplified expression leading to final answer. |
| | 144 | **B1** | WWW |
| | | **4** | |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(c) | **Method 1: If denominator is from 7(a), no denominator or incorrect denominator** | | |
| | [Numerator = Number of required arrangements =]<br>$\frac{5!}{2!}\times\frac{5!}{4!}\times 2$ [ = 600] | **B1** | $\frac{5!}{2!}$ seen (arrangements of consonants). |
| | | **B1** | $\frac{5!}{4!}$ seen (arrangements of vowels). |
| | | **M1** | $\frac{5!}{r}\times\frac{5!}{s}\times 2$, $r$ = 1 or 2, $s$ = 1, 4, 4! or 24. |
| | [Probability =] $\frac{\textit{their}\,600}{\textit{their}\,75600}$ | **M1** | $\frac{\textit{their}\,600}{\textit{their}\,(\mathbf{a})}$ or $\frac{\textit{their}\,600}{75600}$. |
| | = $\frac{1}{126}$, 0.00794 | **A1** | Accept $\frac{600}{75600}$ OE. |
| | **Method 2: If denominator 10!** | | |
| | [Numerator = Number of required arrangements =] | **(B1)** | 5! seen (arrangements of consonants). |
| | $5!\times 5!\times 2$ [ = 28800] | **(B1)** | A second 5! seen (arrangements of vowels). |
| | | **(M1)** | $5!\times 5!\times k$, $k$ = 1 or 2. |
| | [Probability = ] $\frac{\textit{their}\,28800}{10!}$ | **(M1)** | |
| | = $\frac{1}{126}$, 0.00794 | **(A1)** | Accept $\frac{600}{75600}$ OE. |

| Question | Answer | Marks | Guidance |
|---|---|---|---|
| 7(c) | **Method 3: Using probabilities** | | |
| | $\frac{5}{10}\times\frac{5}{9}\times\frac{4}{8}\times\frac{4}{7}\times\frac{3}{6}\times\frac{3}{5}\times\frac{2}{4}\times\frac{2}{3}\times\frac{1}{2}\times\frac{1}{1}\times 2$ | **(B1)** | $\frac{5}{a}\times\frac{4}{b}\times\frac{3}{c}\times\frac{2}{d}\times\frac{1}{e}$ seen.<br>$10 \geq a > b > c > d > e \geq 1$ (arrangements of consonants). |
| | | **(B1)** | A second $\frac{5}{f}\times\frac{4}{g}\times\frac{3}{h}\times\frac{2}{i}\times\frac{1}{j}$ seen.<br>$10 \geq f > g > h > i > j \geq 1$ (arrangements of vowels). |
| | | **(M1)** | $\frac{5}{a}\times\frac{4}{b}\times\frac{3}{c}\times\frac{2}{d}\times\frac{1}{e}\times\frac{5}{f}\times\frac{4}{g}\times\frac{3}{h}\times\frac{2}{i}\times\frac{1}{j}\times k$<br>$k = 1$ or 2. |
| | | **(M1)** | $\frac{\textit{their}\, 5\times4\times3\times2\times1\times5\times4\times3\times2\times1}{10\times9\times8\times7\times6\times5\times4\times3\times2\times1}$. |
| | $= \frac{1}{126}$, 0.00794 | **(A1)** | Accept $\frac{28800}{362800}$ OE. |
| | | **5** | |